

**Basic properties:**

$$d \equiv {}^2_1\text{H}$$

mass:  $mc^2 = 1876.124 \text{ MeV}$

binding energy:  $B \equiv \sum_i m_i - M = m_p + m_n - m_d = 2.2245731 \text{ MeV}$   
(measured via  $\gamma$ -ray energy in  $n + p \rightarrow d + \gamma$ )

RMS radius:  $1.963 \pm 0.004 \text{ fm}$  (from electron scattering)

quantum numbers:  $J^\pi, I = 1^+, 0$  (lectures 13, 14)

magnetic moment:  $\mu = +0.8573 \mu_N$

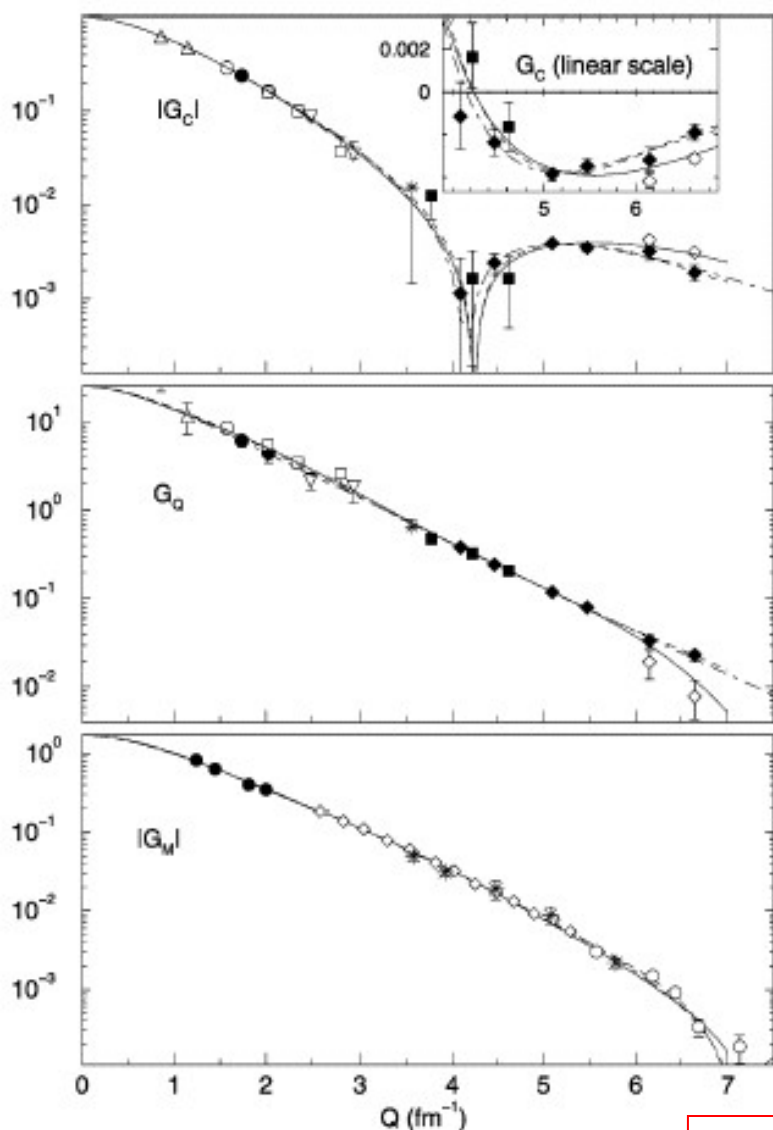
electric quadrupole moment:  $Q = +0.002859 \pm 0.00030 \text{ bn}$   
( $\rightarrow$  the deuteron is not spherical! ....)

**Important because:**

- deuterium is the lightest nucleus and the only bound N-N state
- testing ground for state-of-the art models of the N-N interaction.

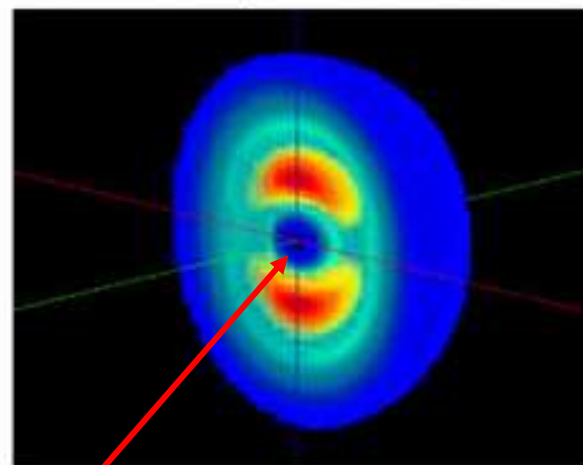
## Electron scattering measurements:

2



Because the deuteron has spin 1, there are 3 form factors to describe elastic scattering: the "charge" ( $G_c$ ), "electric quadrupole" ( $G_q$ ) and "magnetic" ( $G_M$ ) form factors. (JLab data)

Combined Data  $\Rightarrow$   
Intrinsic Shape of the Deuteron



Interesting feature: strong attractive np force, but a void in the center - the deuteron is hollow! ... **Why?**

Interpretation of quantum numbers:  $J^\pi, I = 1^+, 0$

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3

$$\vec{S} \equiv \vec{S}_n + \vec{S}_p$$

$$\vec{J} = \vec{S} + \vec{L} = \vec{1}$$

$$\pi = (+)(+)(-1)^L = + \Rightarrow L = 0, 2, 4 \dots$$

- Of the possible quantum numbers,  $L = 0$  has the lowest energy, so we expect the ground state to be  $L = 0, S = 1$  (the deuteron has no excited states!)
- The nonzero electric quadrupole moment suggests an admixture of  $L = 2$  (more later!)

introduce **Spectroscopic Notation**:  $^{2S+1}L_J$

with naming convention:  $L = 0$  is an S-state,  $L = 1$  is a P-state,  $L = 2$ : D-state, etc...

→ the deuteron configuration is primarily  $^3S_1$

$$\vec{I} = \frac{\vec{1}}{2} + \frac{\vec{1}}{2} \Rightarrow I = 0, 1$$

The total wavefunction for two identical Fermions has to be **antisymmetric** w.r.to particle exchange:

$$\Psi_{total} = \psi_{space} \times \phi_{spin} \times \chi_{isospin}$$

Central force problem:

$$\psi_{space}(r, \theta, \phi) = f(r) Y_{LM}(\theta, \phi)$$

with **symmetry**  $(-1)^L$  given by the spherical harmonic functions

Spin and Isospin configurations:

**$S = 0$  and  $I = 0$  are antisymmetric ;**

**$S = 1$  and  $I = 1$  are symmetric**

**${}^3S_1$  state can only be  $I = 0$  !**

Magnetic Moment:  $\mu_d = 0.857 \mu_N$

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In general, the magnetic moment is a quantum-mechanical vector; it must be aligned along the "natural symmetry axis" of the system, given by the total angular momentum:

$$\vec{\mu} \sim \vec{J}$$

But we don't know the direction of  $\vec{J}$ , only its "length" and z-projection as expectation values:

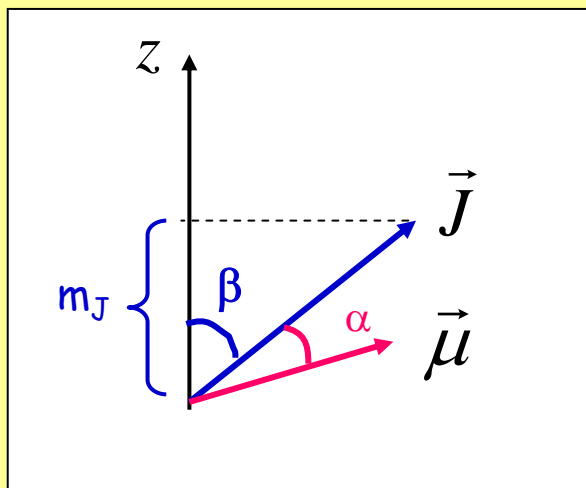
$$\langle J^2 \rangle = J(J+1); \quad \langle J_z \rangle = m_J = (-J \dots + J)$$

in a magnetic field, the energy depends on  $m_J$  via  $\Delta E = -\langle \vec{\mu} \cdot \vec{B} \rangle \equiv -g_J m_J B \mu_N$

Strategy: we will define the magnetic moment by its maximal projection on the z-axis, defined by the direction of the magnetic field, with  $m_J = J$

$$\mu \equiv \langle \vec{\mu} \cdot \hat{z} \rangle \Big|_{m_J=J} = g_J J \mu_N$$

Use expectation values of operators to calculate the result....



$$\mu \equiv \left\langle \vec{\mu} \cdot \hat{z} \right\rangle \Big|_{m_J=J} = g_J J \mu_N$$

Subtle point: we have to make two successive projections to evaluate the magnetic moment according to our definition, and the spin and orbital contributions enter with different weights.

1. Project onto the direction of  $J$ :

$$\vec{\mu} \cdot \hat{J} = \mu \cos \alpha = \frac{\vec{\mu} \cdot \vec{J}}{\sqrt{J(J+1)}}$$

2. Project onto the z-axis with  $m_J = J$ :

$$\cos \beta = \frac{m_J}{|J|} = \frac{J}{\sqrt{J(J+1)}}$$

$$\mu \equiv g_J J \mu_N = \mu \cos \alpha \cos \beta = \left\langle \vec{\mu} \cdot \vec{J} \right\rangle \frac{1}{(J+1)}$$

Next, we need to figure out the operator for  $\vec{\mu}$

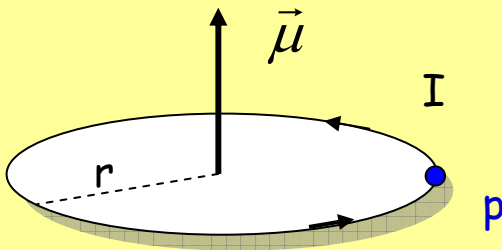
## spin and orbital contributions:

We already know the intrinsic magnetic moments of the proton and neutron, so these must correspond to the spin contributions to the magnetic moment operator:

$$\mu_p = +2.79 \mu_N = g_{s,p} S \mu_N \Rightarrow g_{s,p} = +5.58$$

$$\mu_n = -1.91 \mu_N = g_{s,n} S \mu_N \Rightarrow g_{s,n} = -3.83$$

For the orbital part, there is a contribution from the proton only, corresponding to a circulating current loop (semiclassical sketch, but the result is correct)



$$\vec{\mu} = I \pi r^2 \hat{\ell} = g_\ell \vec{\ell} \mu_N, \quad g_\ell = 1$$

For the deuteron, we want to use the magnetic moment operator:

$$\vec{\mu} = \left( g_{s,p} \vec{S}_p + g_{s,n} \vec{S}_n + \vec{L}_p \right) \mu_N$$

1.  $\vec{L}_p = \frac{1}{2} \vec{L}$  because  $m_n \cong m_p$ , and **L is the total orbital angular momentum !**
2.  $L = 0$  in the "S-state" ( $^3S_1$ ) but we will consider also a contribution from the "D-state" ( $^3D_1$ ) as an exercise
3. The proton and neutron couple to  $S = 1$ , and the deuteron has  $J = 1$

$$\mu = \frac{1}{2} \langle \vec{\mu} \cdot \vec{J} \rangle = \frac{1}{2} \left\langle \left( g_{s,p} \vec{S}_p + g_{s,n} \vec{S}_n + \frac{1}{2} \vec{L} \right) \cdot \vec{J} \right\rangle \mu_N$$

Trick: use  $\vec{S}_n + \vec{S}_p = \vec{S}$  and write the **operator** as:

$$\vec{\mu} = \left( \frac{1}{2} (g_{s,p} + g_{s,n}) \vec{S} + \frac{1}{2} (g_{s,p} - g_{s,n}) (\vec{S}_p - \vec{S}_n) + \frac{1}{2} \vec{L} \right) \mu_N$$

But the proton and neutron spins are aligned, and  $\langle \vec{S}_p \cdot \vec{J} \rangle = \langle \vec{S}_n \cdot \vec{J} \rangle$   
so the second term has to give zero!



So, effectively we can write for the deuteron:

$$\mu = \frac{1}{4} \left\langle \left( (g_{s,p} + g_{s,n}) \vec{S} + \vec{L} \right) \cdot \vec{J} \right\rangle \mu_N$$

Trick for expectation values:

$$\vec{J} = \vec{L} + \vec{S}; \quad \langle J^2 \rangle = J(J+1)$$

$$\langle J^2 \rangle = \langle (\vec{L} + \vec{S}) \cdot (\vec{L} + \vec{S}) \rangle = \langle L^2 + S^2 + 2\vec{L} \cdot \vec{S} \rangle$$

$$\langle \vec{S} \cdot \vec{J} \rangle = \langle \vec{S} \cdot \vec{L} + \vec{S} \cdot \vec{S} \rangle = \frac{1}{2} \langle J^2 - L^2 - S^2 \rangle + \langle S^2 \rangle = \frac{1}{2} \langle J^2 - L^2 + S^2 \rangle$$

$$\langle \vec{L} \cdot \vec{J} \rangle = \langle \vec{L} \cdot \vec{L} + \vec{L} \cdot \vec{S} \rangle = \frac{1}{2} \langle J^2 + L^2 - S^2 \rangle$$



$$\mu(^3S_1) = \frac{1}{2} (g_{s,p} + g_{s,n}) \mu_N = \mu_p + \mu_n = 0.880 \mu_N$$

$$\mu(^3D_1) = \frac{1}{4} \left( 3 - (g_{s,p} + g_{s,n}) \right) \mu_N = 0.310 \mu_N$$

$$\mu_d = 0.857 \mu_N$$

This is intermediate between the S-state and D-state values:

$$\mu(^3S_1) = 0.880 \mu_N$$

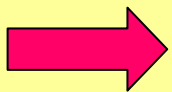
$$\mu(^3D_1) = 0.310 \mu_N$$

Suppose the wave function of the deuteron is a **linear combination** of S and D states:

$$|\psi_d\rangle = a |^3S_1\rangle + b |^3D_1\rangle \quad \text{with} \quad a^2 + b^2 = 1$$

Then we can **adjust the coefficients** to explain the magnetic moment:

$$\mu_d = (1-b^2) \mu(^3S_1) + b^2 \mu(^3D_1)$$



$b^2 = 0.04$ , or a 4% D-state admixture accounts for the magnetic moment !

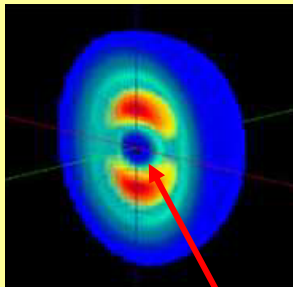
Quantum numbers:  $(J^\pi, I) = (1^+, 0)$  favor a  ${}^3S_1$  configuration (with  $S = 1, L = 0$ ) as the lowest energy n-p bound state

Magnetic moment:  $\mu = 0.857 \mu_N$  is 2.6% smaller than for a pure  ${}^3S_1$  state and is consistent with a linear combination of  $L = 0$  and  $L = 2$  components:

$$|\psi_d\rangle = a |{}^3S_1\rangle + b |{}^3D_1\rangle \quad \text{with} \quad a^2 + b^2 = 1 \quad \text{and} \quad b^2 = 0.04$$

If this is correct, we can draw two conclusions about the N-N force:

deuteron:



1. The  $S = 1$  configuration has lowest energy (i.e., observed deuteron quantum numbers), so **the N-N potential must be more attractive for total spin  $S = 1$  than for  $S = 0$ .**

2. The lowest energy solution for a spherically symmetric potential is purely  $L = 0$ , and the  $L = 2$  wave functions are orthogonal to  $L = 0$  wave functions, so a physical deuteron state with mixed symmetry can only arise if **the N-N potential is not exactly spherically symmetric!**

Also: the "hole" in the middle of the deuteron means that **at very short distances the N-N potential must be repulsive** - the neutron and proton do not overlap!

Formalism for electrostatic moments:

(see also D.J. Griffiths, 'Introduction to Electrodynamics' Ch. 3, and 16.369)

- Recall that an electric charge distribution of arbitrary shape can be described by an infinite series of **multipole moments** (spherical harmonic expansion), with terms of higher L reflecting more complicated departures from spherical symmetry
- The energy of such a distribution interacting with an external electric field  $\vec{E} = -\vec{\nabla}V$  can be written as:

$$E_{\text{int}} = V(0) q + \left. \frac{\partial V}{\partial z} \right|_0 p_z - \frac{1}{4} \left. \frac{\partial^2 V}{\partial z^2} \right|_0 Q_{zz} + \dots$$

where the charge distribution is located at the origin and has a **symmetry axis along z**;

**q** is the electric charge or 'monopole moment'

**p<sub>z</sub>** is the electric dipole moment

**Q<sub>zz</sub>** is the electric quadrupole moment

and in general, higher order multipole moments couple to higher derivatives of the external potential at the origin.

The multipole moments are defined in general by:

$$E_\ell = \int \rho(\vec{r}) \hat{E}_\ell d^3r$$

with corresponding multipole operators proportional to spherical harmonic functions of order  $\ell$ :

$$\hat{E}_\ell = c_\ell r^\ell Y_{\ell 0}(\theta)$$

The first few multipole operators and associated moments are:

order, $\ell$	Moment	Symbol	operator, $\hat{E}_\ell$
0	<i>monopole</i>	$q$	$1 = \sqrt{4\pi} r^0 Y_{00}(\theta)$
1	<i>dipole</i>	$p_z$	$z = r \cos \theta = \frac{\sqrt{4\pi}}{3} r^1 Y_{10}(\theta)$
2	<i>quadrupole</i>	$Q_{zz}$	$r^2 (3 \cos^2 \theta - 1) = \frac{\sqrt{16\pi}}{5} r^2 Y_{20}(\theta)$

For a nuclear system, the multipole moments are **expectation values of the multipole operators**, e.g. the electric charge:

$$q \equiv \int \rho(\vec{r}) d^3r \equiv e \sum_{i=1}^Z \int \psi_i^*(\vec{r}) \hat{E}_0 \psi_i(\vec{r}) d^3r = +Ze$$

The electric charge density is proportional to the probability density, i.e. the wave function squared, summed up for all the protons in the system !!!

We already know the electric charge of the deuteron, but what about higher moments?

Electric dipole moment, general case:

$$p_z = \int \Psi^* \hat{E}_1 \Psi d^3r = \int |\Psi|^2 r \cos\theta d^3r = 0$$

total wave function  
of the system

even under  
space reflection

odd,  
 $\ell = 1$

integral  
**must vanish!**

Basic symmetry property: no quantum system can have an electric dipole moment if its wave function has definite parity, i.e. if  $|\Psi|^2$  is even.

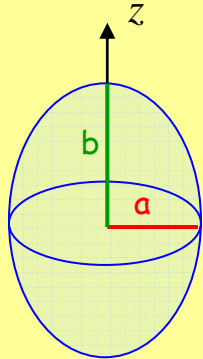
(Incidentally, the possibility of a nonzero electric dipole moment of the **neutron** is of great current interest: so far the measured upper limit is  $|p_n| < 10^{-25} \text{ e} \cdot \text{cm} \dots$ )

In fact, **all odd multipole moments must vanish** for the same reason

First nontrivial case: Electric Quadrupole Moment  $Q_{zz}$

(F&H sec. 18.1)

$$Q_{zz} = \int \rho(\vec{r}) r^2 (3 \cos^2 \theta - 1) d^3 r$$

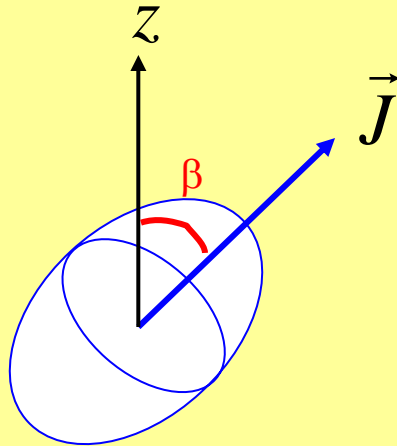


example: uniform density object with ellipsoidal shape

$$Q_{zz} = \frac{2}{5} Ze (b^2 - a^2)$$

$Q > 0$  if  $b > a$ : "prolate"  
 $Q < 0$  if  $b < a$ : "oblate"

Assume that the charge distribution is an ellipsoid of revolution with symmetry axis along the total angular momentum vector  $\vec{J}$ :



As for the magnetic dipole moment, we specify the **intrinsic** electric quadrupole moment as the expectation value when  $J$  is maximally aligned with  $z$ :

$$Q_{\text{int}} = \left\langle \hat{E}_2 \right\rangle \Big|_{m_J=J}$$

"Quantum geometry":

$$\cos \beta = \frac{m_J}{|J|} = \frac{J}{\sqrt{J(J+1)}}$$

If an electric field gradient is applied along the  $z$ -axis as shown, the observable energy will shift by an amount corresponding to the intrinsic quadrupole moment transformed to a coordinate system **rotated through angle  $\beta$**  to align with the  $z$ -axis



Let  $Q_{\text{lab}}$  be the electric quadrupole moment we **measure** for the ellipsoidal charge distribution with  $m_J = J$ :

$$Q_{\text{lab}} = \frac{1}{2} \left( 3 \cos^2 \beta - 1 \right) Q_{\text{int}} = \left( \frac{J - 1/2}{J + 1} \right) Q_{\text{int}}$$

standard, classical  
prescription for  
rotated coordinates

"Quantum geometry"  
for the rotation function

How do we apply this to anything?

1. A spherically symmetric state ( $L = 0$ ) has  $Q_{\text{int}} = 0$  (e.g. deuteron S-state)
  2. Even a distorted state with  $J = \frac{1}{2}$  will not have an observable quadrupole moment
  3.  $J = 1, L = 2$  is the smallest value of total angular momentum for which we can observe a nonzero quadrupole moment
- these are the quantum numbers for the deuteron D-state!

recall our model for the deuteron wave function:

$$|\psi_d\rangle = a |^3S_1\rangle + b |^3D_1\rangle$$

result for the quadrupole moment:

note cancellation here

$$\begin{aligned} Q_{\text{int}} &= \frac{\sqrt{2}}{10} |a^*b| \langle r^2 \rangle_{SD} - \frac{1}{20} b^2 \langle r^2 \rangle_{DD} \\ &= +0.00286 \pm 0.00003 \text{ bn} \end{aligned}$$

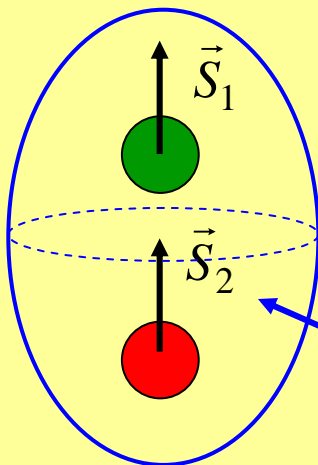
A good model of the N-N interaction can fit both the magnetic moment and the quadrupole moment of the deuteron with the same values of  $a$  and  $b$ !  $\rightarrow$

1. Independent of the value of  $L$ , the state with intrinsic spins coupled to  $S = 1$  has lower energy

→ this implies a term proportional to:

$$-\langle \vec{S}_1 \cdot \vec{S}_2 \rangle = -\frac{1}{2} \langle S^2 - S_1^2 - S_2^2 \rangle = \begin{cases} -1/4, & S=1 \\ +3/4, & S=0 \end{cases}$$

2. The deuteron quadrupole moment implies a non-central component, i.e. the potential is not spherically symmetric. Since the symmetry axis for  $Q$  is along  $J$ ,  $Q > 0$  means that **the matter distribution is stretched out along the  $J$  - axis**:



→ This implies a "tensor" force, proportional to:

$$-\langle S_{12} \rangle = -\left\langle 3 \frac{(\vec{S}_1 \cdot \vec{r})(\vec{S}_2 \cdot \vec{r})}{r^2} - \vec{S}_1 \cdot \vec{S}_2 \right\rangle$$

$Q > 0$ , observed

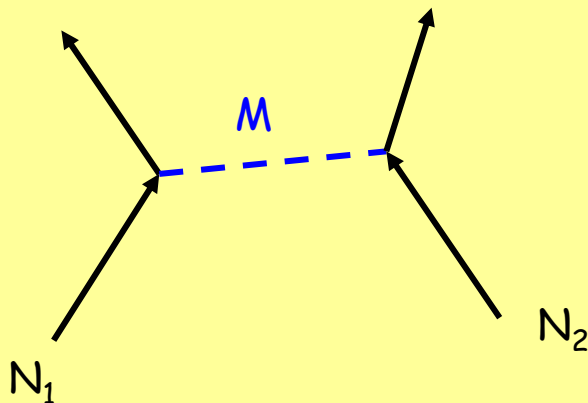
compare: magnetic dipole-dipole interaction, see Griffiths problem 6.20

3. There is also a **spin-orbit term**, as deduced from N-N scattering experiments with a polarized beam:

$$V_{S-O} \sim \langle \vec{L} \cdot \vec{S} \rangle$$

(This plays a very important role also in determining the correct order of energy levels in nuclear spectra - more later!)

4. Finally, all contributions to the N-N interaction are based on a microscopic meson exchange mechanism, as explored in assignment 5:



Where  $M$  is a  $\pi$ ,  $\rho$ ,  $\omega$  ... meson, etc.

and each term has a spatial dependence of the form:

$$V(r) = g \frac{e^{-m r}}{r} \times (\text{spin function})$$

PHYSICS REPORTS (Review Section of Physics Letters) 149, No. 1 (1987) 1–89. North-Holland, Amsterdam

(89 page exposition of one of only ~3 state-of-the-art models of the N-N interaction worldwide - constantly refined and updated since first release.)

## THE BONN MESON-EXCHANGE MODEL FOR THE NUCLEON-NUCLEON INTERACTION\*

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pseudoscalar mesons:

$$V_{ps}(m_{ps}, r) = \frac{1}{12} \frac{g_{ps}^2}{4\pi} m_{ps} \left[ \left( \frac{m_{ps}}{m} \right)^2 Y(m_{ps}r) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + Z(m_{ps}r) S_{12} \right]; \quad (F.6)$$

(tensor operator)

scalar mesons:

$$V_s(m_s, r) = -\frac{g_s^2}{4\pi} m_s \left\{ \left[ 1 - \frac{1}{4} \left( \frac{m_s}{m} \right)^2 \right] Y(m_s r) + \frac{1}{4m^2} [\nabla^2 Y(m_s r) + Y(m_s r) \nabla^2] + \frac{1}{2} Z_1(m_s r) \mathbf{L} \cdot \mathbf{S} \right\}; \quad (F.7)$$

(they use "σ" for spin)

(spin-orbit interaction)

vector mesons:

$$\begin{aligned} V_v(m_v, r) = & \frac{g_v^2}{4\pi} m_v \left\{ \left[ 1 + \frac{1}{2} \left( \frac{m_v}{m} \right)^2 \right] Y(m_v r) - \frac{3}{4m^2} [\nabla^2 Y(m_v r) + Y(m_v r) \nabla^2] \right. \\ & + \frac{1}{6} \left( \frac{m_v}{m} \right)^2 Y(m_v r) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{3}{2} Z_1(m_v r) \mathbf{L} \cdot \mathbf{S} - \frac{1}{12} Z(m_v r) S_{12} \left. \right\} \\ & + \frac{1}{2} \frac{g_v f_v}{4\pi} m_v \left\{ (m_v/m)^2 Y(m_v r) + \frac{2}{3} (m_v/m)^2 Y(m_v r) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \right. \\ & - 4 Z_1(m_v r) \mathbf{L} \cdot \mathbf{S} - \frac{1}{3} Z(m_v r) S_{12} \left. \right\} \\ & + \frac{f_v^2}{4\pi} m_v \left\{ \frac{1}{6} (m_v/m)^2 Y(m_v r) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{1}{12} Z(m_v r) S_{12} \right\}, \end{aligned}$$



with

$$Y(x) = e^{-x/x}, \quad Z(x) = (m_\alpha/m)^2 (1 + 3/x + 3/x^2) Y(x), \quad \text{"Yukawa functions" and derivatives...}$$

$$Z_1(x) = \left(\frac{m_\alpha}{m}\right)^2 (1/x + 1/x^2) Y(x), \quad S_{12} = 3 \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{r})(\boldsymbol{\sigma}_2 \cdot \mathbf{r})}{r^2} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, \quad (\text{F.8})$$

and

$$\nabla^2 = + \frac{1}{r} \frac{d^2}{dr^2} r - \frac{L^2}{r^2}.$$

We use units such that  $\hbar = c = 1$  ( $\hbar c = 197.3286 \text{ MeV fm}$ ). The use of the form factor, eq. (3.3), at each vertex (with  $n_\alpha = 1$ ) leads to the following extended expressions:

$$V_\alpha(r) = V_\alpha(m_\alpha, r) - \frac{\Lambda_{\alpha,2}^2 - m_\alpha^2}{\Lambda_{\alpha,2}^2 - \Lambda_{\alpha,1}^2} V_\alpha(\Lambda_{\alpha,1}, r) + \frac{\Lambda_{\alpha,1}^2 - m_\alpha^2}{\Lambda_{\alpha,2}^2 - \Lambda_{\alpha,1}^2} V_\alpha(\Lambda_{\alpha,2}, r), \quad (\text{F.9})$$

where  $\Lambda_{\alpha,1} = \Lambda_\alpha + \varepsilon$ ,  $\Lambda_{\alpha,2} = \Lambda_\alpha - \varepsilon$ ,  $\varepsilon/\Lambda_\alpha \ll 1$ .  $\varepsilon = 10 \text{ MeV}$  is an appropriate choice.

The full NN potential is the sum of the contributions from six mesons:

$$V(r) = \sum_{\alpha=\pi,\rho,\eta,\omega,\delta,\sigma} V_\alpha(r)$$

*R. Machleidt et al., The Bonn meson-exchange model for the nucleon–nucleon interaction*

Table 14

Meson and low-energy parameters (LEP) for the configuration space one-boson-exchange potential (OBEPR)

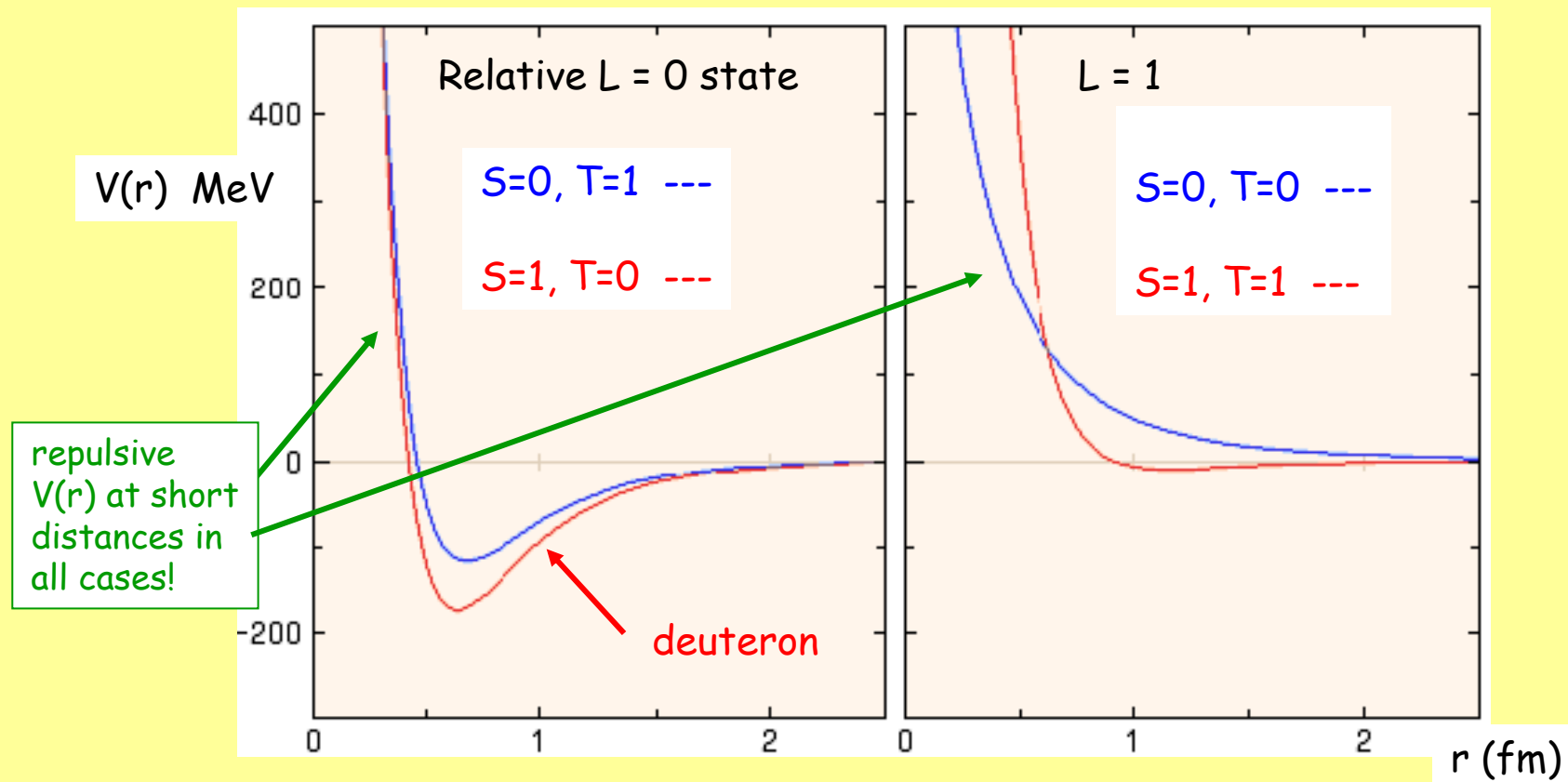
	$g_\alpha^2/4\pi; [f_\alpha/g_\alpha]$	$m_\alpha$ (MeV)	$\Lambda_\alpha$ (GeV)	deuteron properties:	
				LEP	Theory
$\pi$	14.9	138.03	1.3	$\epsilon_d$ (MeV)	2.2246
				$P_D$ (%)	4.81
$\rho$	0.95; [6.1]	769	1.3	$Q_d$ (fm <sup>2</sup> )	0.274
				$\mu_d$ ( $\mu_N$ )	0.8524
$\eta$	3	548.8	1.5	$A_S$ (fm <sup>-1/2</sup> )	0.8860
				D/S	0.0260
$\omega$	20; [0.0]	782.6	1.5	$r_d$ (fm)	1.9691
				$a_s$ (fm)	-23.751
$\delta$	2.6713	983	2.0	$r_s$ (fm)	2.662
				$a_t$ (fm)	5.423
$\sigma$	7.7823 <sup>a</sup>	550 <sup>a</sup>	2.0	$r_t$ (fm)	1.759

Impressively good agreement for ~ 10,000 experimental data points in assorted n-p and p-p scattering experiments plus deuteron observables:  $\chi^2/d.f \sim 1.07$  !

low energy scattering parameters, etc:



It looks different in different spectroscopic states of the 2N system!



Only the deuteron is bound! Its quantum numbers have the deepest potential well.